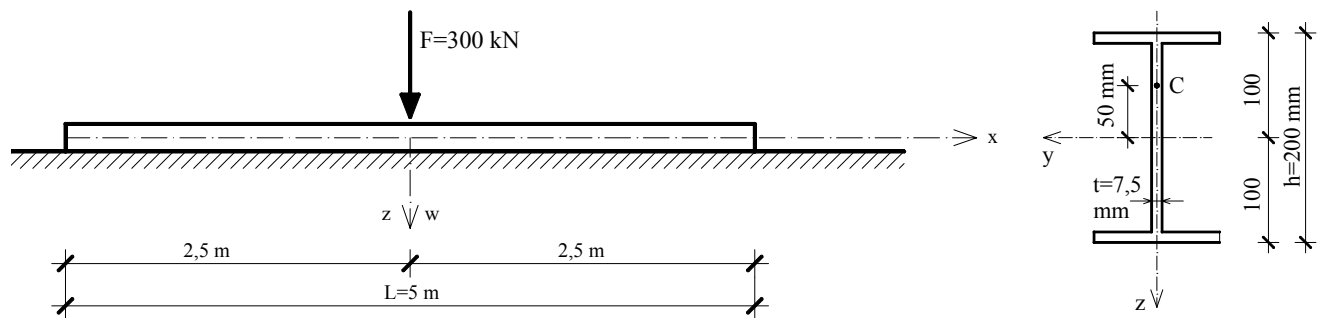


Za zadani nosač položen na elastičnu podlogu, prikazan na slici, treba odrediti progibnu liniju nosača te dijagrame unutrašnjih sila. Nadalje, treba odrediti najveća normalna i najveća posmična naprezanja u nosaču te glavna naprezanja u točki „C“ presjeka $x=0$.

Zadani su: koeficijent podloge $k = 900 \text{ MPa}$, modul elastičnosti $E = 2,1 \cdot 10^5 \text{ MPa}$, aksijalni moment tromosti $I_y = 2,14 \cdot 10^7 \text{ mm}^4$, statički moment površine polovice presjeka

$$S_y = 1,17 \cdot 10^5 \text{ mm}^3$$



Parametar α koji ovisi o odnosu krutosti elastične podloge i nosača:

$$\alpha = \sqrt[4]{\frac{k}{4 \cdot E \cdot I_y}} = \sqrt[4]{\frac{900}{4 \cdot 2,1 \cdot 10^5 \cdot 2,14 \cdot 10^7 \cdot 10^{-12}}} = 2,66 \frac{1}{m}$$

$$\alpha \cdot L = 2,66 \cdot 5 = 13,3 > 5 \rightarrow \text{nosač se može promatrati kao beskonačno dug}$$

Jednadžba elastične linije nosača:

$$w = \frac{F}{8 \cdot E \cdot I_y \cdot \alpha^3} \cdot e^{-\alpha x} \cdot [\cos(\alpha x) + \sin(\alpha x)]$$

Jednadžba za kutove zaokreta:

$$\varphi = w' = -\frac{F}{4 \cdot E \cdot I_y \cdot \alpha^2} \cdot e^{-\alpha x} \cdot \sin(\alpha x)$$

Jednadžba za momente savijanja:

$$M = -E \cdot I_y \cdot w'' = \frac{F}{4 \cdot \alpha} \cdot e^{-\alpha x} \cdot [\cos(\alpha x) - \sin(\alpha x)]$$

Jednadžba za poprečne sile:

$$T(x) = -E \cdot I_y \cdot w''' = -\frac{F}{2} \cdot e^{-\alpha x} \cdot \cos(\alpha x)$$

Da bismo odredili tražene dijagrame, odredimo vrijednosti u karakterističnim presjecima:

Progibna linija

$$\text{Za } x=0 \quad w = \frac{F}{8 \cdot E \cdot I_y \cdot \alpha^3} = 0,433 \text{ mm}$$

Nultočke progibne linije (točke u kojima je progib 0):

$$w = 0 \Rightarrow \cos(\alpha x) + \sin(\alpha x) = 0 \Rightarrow \operatorname{tg}(\alpha x) = -1 \Rightarrow \alpha x_0 = \frac{3}{4}\pi, \frac{7}{4}\pi, \frac{11}{4}\pi \dots$$

$$x_0 = \frac{3 \cdot \pi}{4 \cdot \alpha} = 0,885 \text{ m}$$

$$x_0 = \frac{7 \cdot \pi}{4 \cdot \alpha} = 2,066 \text{ m}$$

$$x_0 = \frac{11 \cdot \pi}{4 \cdot \alpha} = 3,246 \text{ m}$$

Nagib tangente na progibnu liniju

$$\text{Za } x=0 \quad \varphi = w' = 0$$

$$\text{Nultočke: } \varphi = 0 \Rightarrow \sin(\alpha x) = 0 \Rightarrow \alpha x_0 = 0, \pi, 2\pi, 3\pi \dots$$

$$x_0 = 0$$

$$x_0 = \frac{\pi}{\alpha} = 1,180 \text{ m}$$

$$x_0 = \frac{2\pi}{\alpha} = 2,360 \text{ m}$$

$$x_0 = \frac{3\pi}{\alpha} = 3,541 \text{ m}$$

Moment savijanja

$$\text{Za } x=0 \quad M = M_{\max} = \frac{F}{4 \cdot \alpha} = 28,195 \text{ kNm}$$

$$\text{Nultočke: } M = 0 \Rightarrow \cos(\alpha x) - \sin(\alpha x) = 0 \Rightarrow \operatorname{tg}(\alpha x) = 1 \Rightarrow \alpha x_0 = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \dots$$

$$x_0 = \frac{\pi}{4\alpha} = 0,295 \text{ m}$$

$$x_0 = \frac{5\pi}{4\alpha} = 1,475 \text{ m}$$

$$x_0 = \frac{9\pi}{4\alpha} = 2,656 \text{ m}$$

Poprečna sila

$$\text{Za } x=0 \quad T = \frac{-F}{2} ; |T_{\max}| = \frac{F}{2} = 150 \text{ kN}$$

$$\text{Nultočke: } T = 0 \Rightarrow \cos(\alpha x) = 0 \Rightarrow \alpha x_0 = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$$

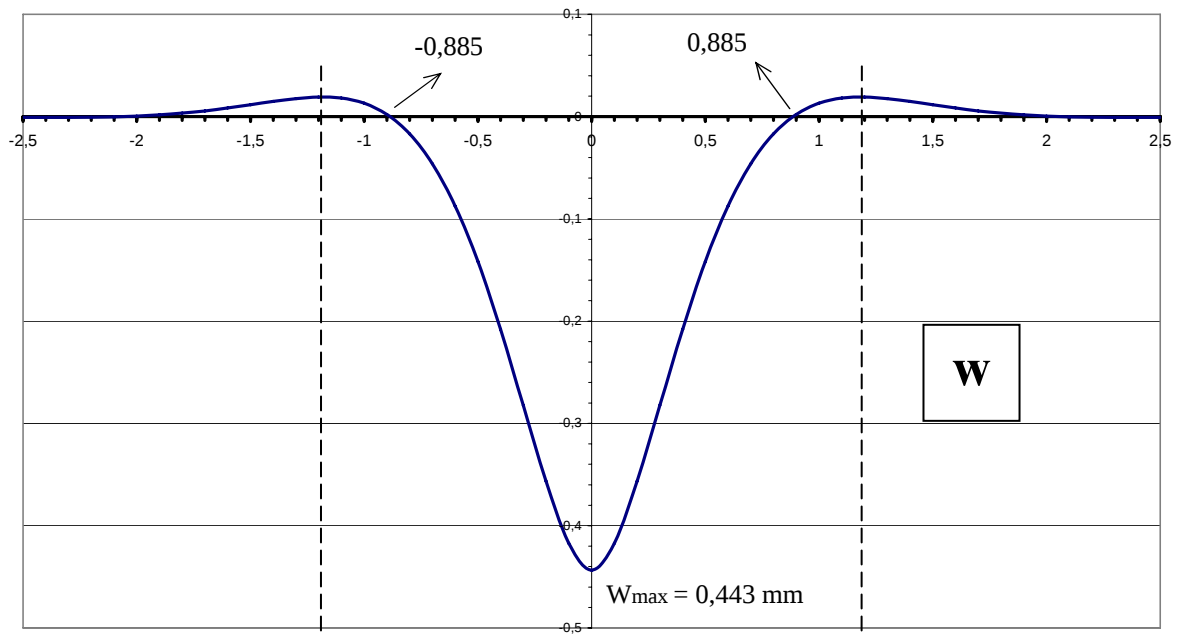
$$x_0 = \frac{\pi}{2\alpha} = 0,590 \text{ m}$$

$$x_0 = \frac{3\pi}{2\alpha} = 1,771 \text{ m}$$

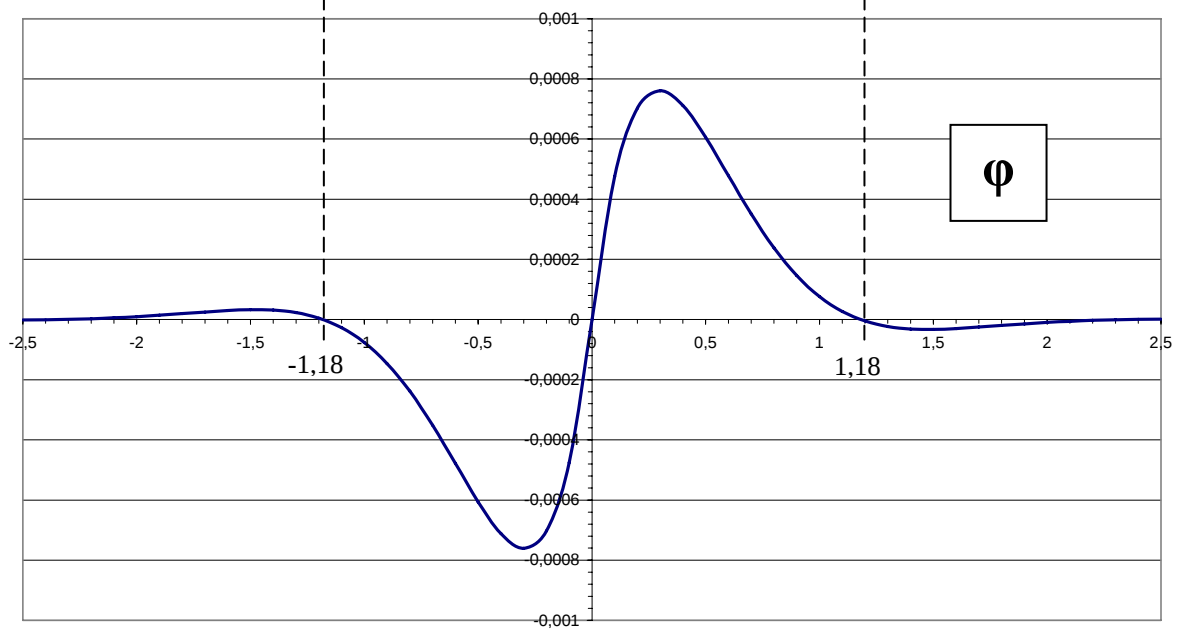
$$x_0 = \frac{5\pi}{2\alpha} = 2,951 \text{ m}$$

Gore navedene jednačbe vrijede za $x \geq 0$. Za dio za koji je $x \leq 0$ koristit ćemo činjenicu da su funkcije progiba i momenta simetrične a kuta zaokreta i poprečne sile antimetrične.

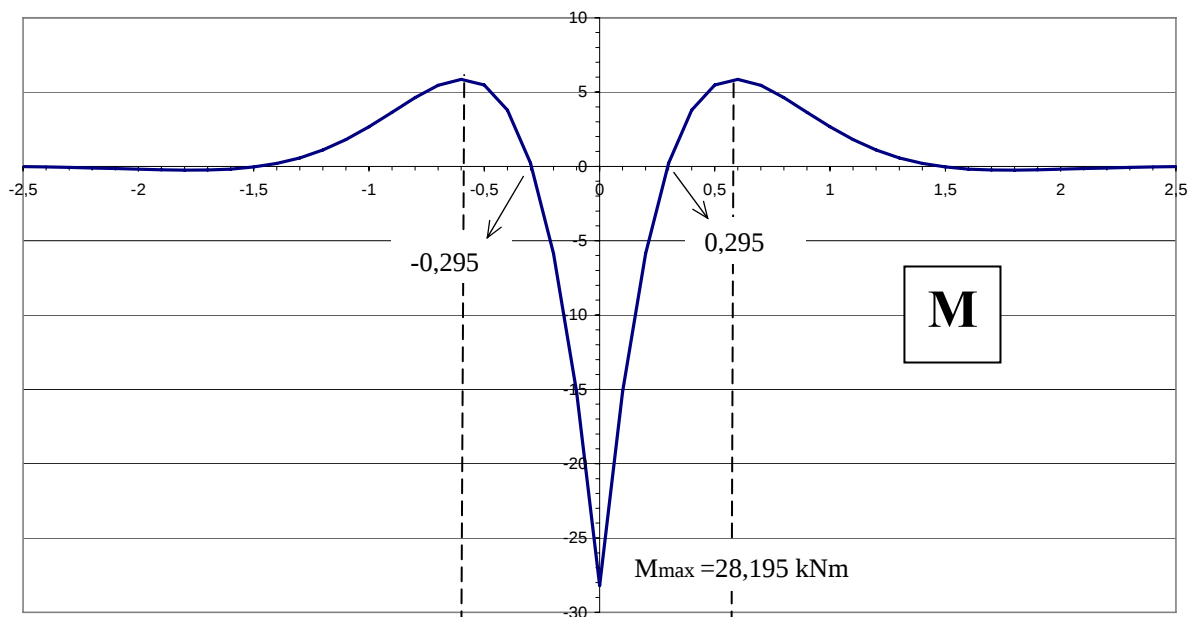
Progibi



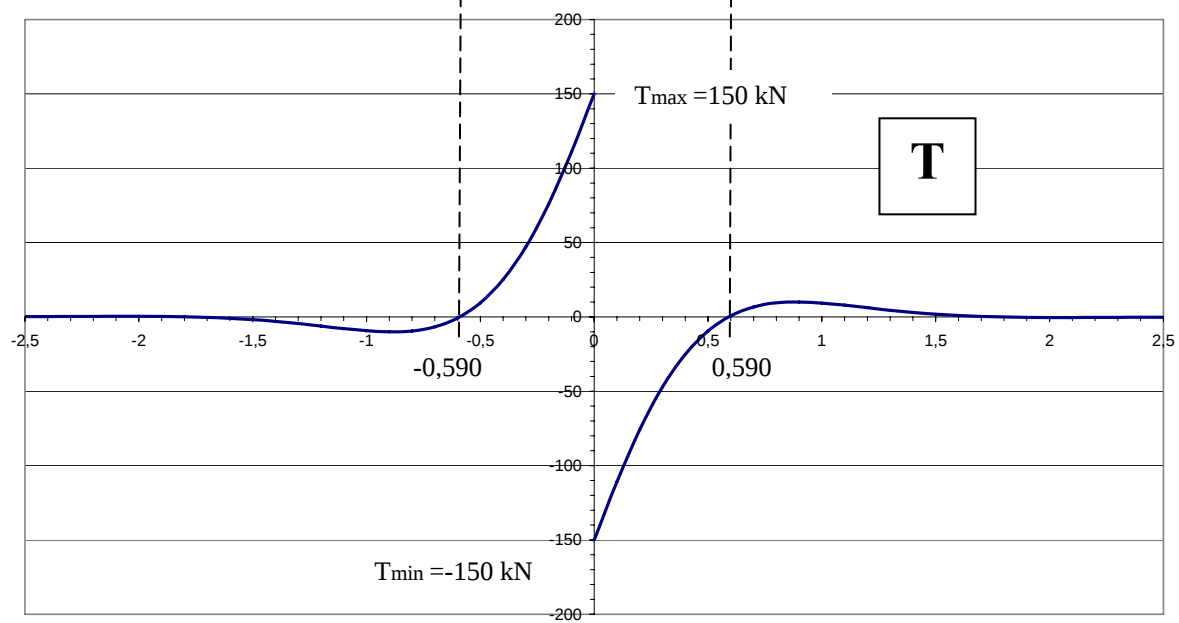
Kut zaokreta



Moment sile



Poprečna sila



Najveća normalna naprezanja:

$$\sigma_{\max} = \frac{M_{\max}}{I_y} \cdot \frac{h}{2} = 131,75 \text{ MPa}$$

Najveća posmična naprezanja:

$$\tau_{\max} = \frac{T_{\max} \cdot S_y}{I_y \cdot t} = 109,35 \text{ MPa}$$

Glavna naprezanja u točki „C“, desno od sile F:

$$\sigma_{x(C)} = \frac{M_{\max}}{I_y} \cdot z_C = -65,88 \text{ MPa}$$

$$\tau_{xz(C)} = \frac{T_C \cdot S_{y(C)}}{I_y \cdot t}; \quad S_{y(C)} = S_y - 7,5 \cdot 50 \cdot \frac{50}{2} = 1,076 \cdot 10^5 \text{ mm}^3$$

$$\tau_{xz(C)} = -100,56 \text{ MPa}$$

$$\sigma_{1,2}^C = \frac{\sigma_x^C}{2} \pm \frac{1}{2} \cdot \sqrt{(\sigma_x^C)^2 + 4 \cdot (\tau_{xz}^C)^2} = -32,94 \pm 105,82 \text{ (MPa)}$$

$$\sigma_1^C = +72,88 \text{ MPa}$$

$$\sigma_2^C = -138,76 \text{ MPa}$$

$$\text{Kontrola: } \sigma_1^C + \sigma_2^C = \sigma_x^C + \sigma_y^C \quad (\sigma_y^C = 0)$$

$$\text{Smjer glavnih naprezanja: } \operatorname{tg}(2\varphi_o) = \frac{2 \cdot \tau_{xz}^C}{\sigma_x^C} = +3,0526 \Rightarrow \varphi_o = +35,93^\circ$$

$$\operatorname{tg} \varphi_{01} = \frac{\tau_{xz}^C}{\sigma_1^C - \sigma_y^C} = -1,380 \Rightarrow \varphi_{01} = -54,07^\circ$$

$$\operatorname{tg} \varphi_{02} = \frac{\tau_{xz}^C}{\sigma_2^C - \sigma_y^C} = +0,724 \Rightarrow \varphi_{02} = +35,93^\circ$$

$$\text{Kontrola: } |\varphi_{01}| + |\varphi_{02}| = 90^\circ$$

